Quantum Gauge Theory of Gravity

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Abstract

The quantum gravity is formulated based on principle of local gauge invariance. The model discussed in this paper has local gravitational gauge symmetry and gravitational field is represented by gauge field. In leading order approximation, it gives out classical Newton's theory of gravity. It can also give out an Einstein-like field equation with cosmological constant. For classical tests, it gives out the same theoretical predictions as those of general relativity. This quantum gauge theory of gravity is a renormalizable quantum theory.

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1 Introduction

Gravity is an ancient topic in science. In ancient times, human has known the existence of weight, which is the gravity between an object and earth. In 1686, Isaac Newton published his epoch-making book *MATHEMATICAL PRINCIPLES OF NATURAL PHILOSOPHY*. Through studying the motion of planet in solar system, he found that the gravity obeys the inverse square law[1]. The Newton's classical theory of gravity is kept unchanged until 1916. At that year, Einstein published his epoch-making paper on General Relativity[2, 3]. In this great work, he founded a relativistic theory on gravity, which is based on principle of general relativity and equivalence principle. Newton's classical theory for gravity appears as a classical limit of general relativity.

1954, Yang and Mills proposed non-Abel gauge field theory[4]. This theory was soon applied to elementary particle physics. Unified electroweak theory[5, 6, 7] and quantum chromodynamics are all based on gauge field theory. Now it is generally believed that four kinds of fundamental interactions in Nature are all gauge interactions and they can be described by gauge field theory. From theoretical point of view, the principle of local gauge invariance plays a fundamental role in particle's interaction theory.

In 1916, Albert Einstein points out that quantum effects must lead to modifications in the theory of general relativity[8]. Soon after the foundation of quantum mechanics, physicists try to found a theory that could describe the quantum behavior of the full gravitational field. In the 70 years attempts, physicists have founded two theories based on quantum mechanics that attempt to unify general relativity and quantum mechanics, one is canonical quantum gravity and another is superstring theory. But for quantum field theory, there are different kinds of mathematical infinities that naturally occur in quantum descriptions of fields. These infinities should be removed by the technique of perturbative renormalization. However, the perturbative renormalization does not work for the quantization of Einstein's theory of gravity, especially in canonical quantum gravity. In superstring theory, in order to make perturbative renormalization to work, physicists have to introduce six extra dimensions. But up to now, none of the extra dimensions have been observed. To found a consistent theory that can unify general relativity and quantum mechanics is a long dream for physicists.

Gauge treatment of gravity was suggested immediately after the gauge theory birth itself[9, 10, 11, 12, 13, 14]. In the traditional gauge treatment of gravity, Lorentz group is localized, and the gravitational fields are not represented by gauge potentials[15, 16, 17]. The theory has beautiful mathematical forms, but it is considered to be non-renormalizable.

2 Gauge Principle

In this paper, we will use completely new notion and completely new method to study quantum gravity. Our goal is to set up a consistent quantum gauge theory of gravity which is renormalizable. The foundation of the new quantum gauge theory of gravity is gauge principle. Gauge principle can be formulated as follows: Any kind of fundamental interactions has a gauge symmetry corresponding to it; the gauge symmetry completely determines the forms of interactions. In principle, gauge principle has the following four different contents:

1. **Conservation Law:** the global gauge symmetry gives out conserved current and conserved charge;

2. **Interactions:** the requirement of the local gauge symmetry requires introduction of gauge field or a set of gauge fields; the interactions between gauge fields and matter fields are completely determined by the requirement of local gauge symmetry; these gauge fields transmit the corresponding interactions;

3. **Source:** qualitative speaking, the conserved charge given by global gauge symmetry is the source of gauge field; for non-Abel gauge field, gauge field is also the source of itself;

4. **Quantum Transformation:** the conserved charges given by global gauge symmetry become generators of quantum gauge transformations after quantization, and for this kind of of interactions, the quantum transformations can not have generators other than quantum conserved charges given by global gauge symmetry.

Gauge principle tells us how to study conservation law and fundamental interactions through symmetry.

It is known that the source of gravitational interactions is energy-momentum, and energy-momentum is the conserved charge of global space-time translation. According to gauge principle, space-time translation group is the symmetry group corresponding to gravity. But the traditional space-time translation group consists of those transformations that objects and fields are kept unchanged while space-time coordinates undergo some transformations. In gravitational gauge theory, we need another kind of translation, that is, space-time coordinates are kept unchanged while objects and fields undergo some transformations. This kind of transformation is called gravitational gauge transformation. From mathematical point of view, gravitational gauge transformation is the inverse transformation of space-time translation, so they are essentially the same. But from physical point of view, they are not the same, especially when we discuss gravitational gauge field. In a meaning, space-time translation is a kind of mathematical transformation; while gravitational gauge transformation is a kind of physical transformation, which contains dynamical information of interactions.

In traditional gauge theory of gravity, Lorentz group is localized. The generators of Lorentz group are angular momentum tensor $M_{\mu\nu}$. According to gauge principle, if we localize Lorentz group, the theory will contain spin-spin interactions, which do not belong to traditional Newton-Einstein gravity. It is known that a theory which contain direct spin-spin interactions is not renormalizable. On the other hand, we do not find any direct evidence on the existence of the direct spin-spin interactions in Nature. For these reasons, we will not localize Lorentz group in this paper. We only localize gravitational gauge group, which is enough for us to set up a consistent quantum theory of gravity. Besides, the gravitational field is represented by gauge potential, and space-time is kept flat. But, if we go into the gravitational gauge group space, we will find that this space-time is curved. However, foundations of gauge theory of gravity is not formulated in this space.

3 Gravitational Interactions of Pure Gravitational Gauge Field

Gravitational gauge group is a transformation group which consists of all non-singular translation operators $\hat{U}_\epsilon$, where $\hat{U}_\epsilon$ is given by

$$\hat{U}_\epsilon \stackrel{\triangle}{=} E^{-i\epsilon^\mu \cdot \hat{P}_\mu}, \qquad \hat{P}_\mu = -i\frac{\partial}{\partial x^\mu}. \tag{3.1}$$

In eq.(3.1), $E^{a^\mu \cdot b_\mu}$ is a special exponential function whose definition is

$$E^{a^\mu \cdot b_\mu} \stackrel{\triangle}{=} 1 + \sum_{n=1}^{\infty} \frac{1}{n!} a^{\mu_1} \cdots a^{\mu_n} \cdot b_{\mu_1} \cdots b_{\mu_n}. \tag{3.2}$$

The gravitational gauge transformation of scalar field and vector field respectively are,

$$\phi(x) \to \phi'(x) = (\hat{U}_\epsilon \phi(x)). \tag{3.3}$$

$$A_\mu(x) \to A'_\mu(x) = (\hat{U}_\epsilon A_\mu(x)). \tag{3.4}$$

A vector field can be a scalar or a vector or a tensor in the space of gauge group. If $A_\mu(x)$ is a vector in the space of gravitational gauge group, it can be expanded as:

$$A_\mu(x) = A_\mu^\alpha(x) \cdot \hat{P}_\alpha, \tag{3.5}$$

where index α is group index and index μ is ordinary Lorentz index. The transformation of component field is

$$A_\mu^\alpha(x) \to A'^\alpha_\mu(x) = \Lambda^\alpha{}_\beta \hat{U}_\epsilon A_\mu^\beta(x) \hat{U}_\epsilon^{-1}, \tag{3.6}$$

where $\Lambda^\alpha{}_\beta$ is given by

$$\Lambda^\alpha{}_\beta = \frac{\partial x^\alpha}{\partial(x-\epsilon(x))^\beta}, \tag{3.7}$$

$A_\mu(x)$ can also be a nth order tensor in the space of gravitational gauge group. Under gravitational gauge transformation, the behavior of a group index is quite different from that of a Lorentz index. However, they have the same behavior under global Lorentz transformation.

Define matrix G as

$$G = (G_\mu^\alpha) = (\delta_\mu^\alpha - gC_\mu^\alpha), \tag{3.8}$$

where C_μ^α is the gravitational gauge field which will be introduced below. A simple form for matrix G is

$$G = I - gC, \tag{3.9}$$

where I is a unit matrix and $C = (C_\mu^\alpha)$. Therefore,

$$G^{-1} = \frac{1}{I - gC}. \tag{3.10}$$

G^{-1} is the inverse matrix of G, it satisfies

$$(G^{-1})_\beta^\mu G_\mu^\alpha = \delta_\beta^\alpha, \tag{3.11}$$

$$G_\mu^\alpha (G^{-1})_\alpha^\nu = \delta_\mu^\nu. \tag{3.12}$$

Define

$$g^{\alpha\beta} \stackrel{\triangle}{=} \eta^{\mu\nu} G^{\alpha}_{\mu} G^{\beta}_{\nu}, \tag{3.13}$$

$$g_{\alpha\beta} \stackrel{\triangle}{=} \eta_{\mu\nu} (G^{-1})^{\mu}_{\alpha} (G^{-1})^{\nu}_{\beta}. \tag{3.14}$$

It can be easily proved that

$$g_{\alpha\beta} g^{\beta\gamma} = \delta^{\gamma}_{\alpha}, \tag{3.15}$$

$$g^{\alpha\beta} g_{\beta\gamma} = \delta^{\alpha}_{\gamma}. \tag{3.16}$$

Under gravitational gauge transformations, they transform as

$$g_{\alpha\beta}(x) \to g'_{\alpha\beta}(x') = \Lambda_{\alpha}^{\ \alpha_1} \Lambda_{\beta}^{\ \beta_1} (\hat{U}_{\epsilon} g_{\alpha_1\beta_1}(x)), \tag{3.17}$$

$$g^{\alpha\beta}(x) \to g'^{\alpha\beta}(x') = \Lambda^{\alpha}_{\ \alpha_1} \Lambda^{\beta}_{\ \beta_1} (\hat{U}_{\epsilon} g^{\alpha_1\beta_1}(x)), \tag{3.18}$$

The gravitational gauge covariant derivative is defined by

$$D_{\mu} = \partial_{\mu} - i g C_{\mu}(x), \tag{3.19}$$

where $C_{\mu}(x)$ is the gravitational gauge field and g is the gravitational gauge coupling constant. It is a Lorentz vector. Under gravitational gauge transformation, $C_{\mu}(x)$ transforms as

$$C_{\mu}(x) \to C'_{\mu}(x) = \hat{U}_{\epsilon}(x) C_{\mu}(x) \hat{U}_{\epsilon}^{-1}(x) + \frac{i}{g} \hat{U}_{\epsilon}(x) (\partial_{\mu} \hat{U}_{\epsilon}^{-1}(x)), \tag{3.20}$$

and D_{μ} transforms covariantly,

$$D_{\mu}(x) \to D'_{\mu}(x) = \hat{U}_{\epsilon} D_{\mu}(x) \hat{U}_{\epsilon}^{-1}. \tag{3.21}$$

Gravitational gauge field $C_{\mu}(x)$ is a vector field, it is a Lorentz vector. It is also a vector in gauge group space, so it can be expanded as linear combinations of generators of gravitational gauge group:

$$C_{\mu}(x) = C^{\alpha}_{\mu}(x) \cdot \hat{P}_{\alpha}. \tag{3.22}$$

C^{α}_{μ} is component field of gravitational gauge field. It looks like a second rank tensor, but it is not tensor field. The index α is not an ordinary Lorentz index, it is just a gauge group index. Gravitational gauge field C^{α}_{μ} has only one Lorentz index, so it is a kind of vector field.

The strength of gravitational gauge field is defined by

$$F_{\mu\nu} = \frac{1}{-ig}[D_\mu \quad , \quad D_\nu], \tag{3.23}$$

or

$$F_{\mu\nu} = \partial_\mu C_\nu(x) - \partial_\nu C_\mu(x) - igC_\mu(x)C_\nu(x) + igC_\nu(x)C_\mu(x). \tag{3.24}$$

$F_{\mu\nu}$ is a second rank Lorentz tensor. It is a vector is group space, so it can be expanded in group space,

$$F_{\mu\nu}(x) = F^\alpha_{\mu\nu}(x) \cdot \hat{P}_\alpha. \tag{3.25}$$

The explicit form of component field strength is

$$F^\alpha_{\mu\nu} = \partial_\mu C^\alpha_\nu - \partial_\nu C^\alpha_\mu - gC^\beta_\mu \partial_\beta C^\alpha_\nu + gC^\beta_\nu \partial_\beta C^\alpha_\mu \tag{3.26}$$

Under gravitational gauge transformation, The component strength of gravitational gauge field transforms as

$$F^\alpha_{\mu\nu} \to F'^\alpha_{\mu\nu} = \Lambda^\alpha{}_\beta(\hat{U}_\epsilon F^\beta_{\mu\nu}). \tag{3.27}$$

Similar to traditional gauge field theory, the kinematical term for gravitational gauge field can be selected as

$$\mathcal{L}_0 = -\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma} g_{\alpha\beta} F^\alpha_{\mu\nu} F^\beta_{\rho\sigma}, \tag{3.28}$$

where $\eta^{\mu\rho}$ is the Minkowski metric. Using eq.(3.17) and eq.(3.27). We can easily prove that this Lagrangian does not invariant under gravitational gauge transformation, it transforms covariantly

$$\mathcal{L}_0 \to \mathcal{L}'_0 = (\hat{U}_\epsilon \mathcal{L}_0). \tag{3.29}$$

In order to resume the gravitational gauge symmetry of the action, we introduce an important factor, which is denoted as $J(C)$. The form of $J(C)$ is not unique[18]. In this paper, $J(C)$ is selected to be

$$J(C) = \sqrt{-\det(g_{\alpha\beta})}, \tag{3.30}$$

where $g_{\alpha\beta}$ is given by eq.(3.14). Using eq.(3.17), we can prove that under gravitational gauge transformation, $J(C)$ transforms as

$$J(C) \to J'(C') = J \cdot (\hat{U}_\epsilon J(C)), \tag{3.31}$$

where

$$J \triangleq \det(\Lambda_\alpha{}^\beta) = \det\left(\frac{\partial(x-\epsilon)^\mu}{\partial x^\nu}\right), \tag{3.32}$$

which is the Jacobian of the corresponding transformation[18]. The Lagrangian for gravitational gauge field is selected as

$$\mathcal{L} \triangleq J(C) \cdot \mathcal{L}_0, \tag{3.33}$$

and the action for gravitational gauge field is

$$S = \int \mathrm{d}^4 x \mathcal{L}. \tag{3.34}$$

Using the following identity

$$\int \mathrm{d}^4 x J \cdot (\hat{U}_\epsilon f(x)) = \int \mathrm{d}^4 x f(x), \tag{3.35}$$

and eq.(3.29) and eq.(3.31), we can prove that this action has local gravitational gauge symmetry.

According to gauge principle, the global symmetry gives out conserved current, it is

$$T^\mu_{i\alpha} \triangleq J(C)\left(-\frac{\partial \mathcal{L}_0}{\partial \partial_\mu C^\beta_\nu}\partial_\alpha C^\beta_\nu + \delta^\mu_\alpha \mathcal{L}_0\right). \tag{3.36}$$

We call it inertial energy-momentum tensor. The equation of motion of gravitational gauge field is

$$\partial_\mu(\eta^{\mu\lambda}\eta^{\nu\tau}g_{\alpha\beta}F^\beta_{\lambda\tau}) = -gT^\nu_{g\alpha}. \tag{3.37}$$

$T^\nu_{g\alpha}$ is also a conserved current. We call it gravitational energy-momentum tensor, which is the source of gravitational gauge field. Its explicit form is

$$\begin{aligned} T^\nu_{g\alpha} \quad = \quad & -\tfrac{\partial \mathcal{L}_0}{\partial D_\nu C^\beta_\mu}\partial_\alpha C^\beta_\mu + G^{-1\nu}_\alpha \mathcal{L}_0 - G^{-1\lambda}_\sigma(\partial_\mu C^\sigma_\lambda)\tfrac{\partial \mathcal{L}_0}{\partial \partial_\mu C^\alpha_\nu} \\ & -\tfrac{1}{2}\eta^{\mu\rho}\eta^{\lambda\sigma}g_{\alpha\beta}G^{-1\nu}_\gamma F^\gamma_{\mu\lambda}F^\beta_{\rho\sigma} + \partial_\mu(\eta^{\nu\lambda}\eta^{\sigma\tau}g_{\alpha\beta}F^\beta_{\lambda\tau}C^\mu_\sigma). \end{aligned} \tag{3.38}$$

Now, we have obtained two different energy-momentum tensors, one is inertial energy-momentum tensor, another is gravitational energy-momentum tensor. The conserved charge given by inertial energy-momentum tensor is the inertial energy-momentum, and the conserved charge given by gravitational energy-momentum tensor is the gravitational energy-momentum.

This quantum gauge theory of gravity is a renormalizable quantum theory. A detailed and strict proof on the renormalizability of the theory can be found in ref. [18]. We will not discuss this problem here.

4 Classical Limit of the Theory

Now, let's see the classical limit of the above theory. Suppose that the gravitational field is static and weak, and the moving speeds of all objects are slow. Then, in leading order approximation, eq.(3.37) gives out

$$\nabla^2 C_0^0 = -g T_0^0. \tag{4.1}$$

Suppose that the source of gravitational field is a point object, that is

$$T_0^0 = -M\delta(\vec{x}). \tag{4.2}$$

From eq.(4.1), we can obtain that

$$g C_0^0 = -\frac{g^2 M}{4\pi r}. \tag{4.3}$$

This is just the gravitational potential which is expected in Newton's theory of gravity. Therefore, in leading order approximation, this theory gives out classical Newton's theory of gravity.

5 Einstein-like Field Equation With Cosmological Constant

In the above chapters, the quantum gravity is formulated in the traditional framework of quantum field theory, i.e., the physical space-time is always flat and the space-time metric is always selected to be the Minkowski metric. In this picture, gravity is treated as physical interactions in flat physical space-time. Our gravitational gauge transformation does not act on physical space-time coordinates, but act on physical fields, so gravitational gauge transformation does not affect the structure of physical space-time. This is one picture of gravity, or call it one representation of gravity theory. For the sake of simplicity, we call it physical representation of gravity.

There is another representation of gravity which is widely used in Einstein's general relativity. This representation is essentially a geometrical representation of gravity. For gravitational gauge theory, if we treat G^{α}_{μ} (or $G^{-1\mu}_{\alpha}$) as a fundamental physical input of the theory, we can also set up the geometrical representation of gravity[20]. For gravitational gauge theory, the geometrical nature of gravity essentially originates from the geometrical nature of the gravitational gauge transformation. In the geometrical picture of gravity, gravity is not treated as a kind of physical interactions, but is treated as the geometry of space-time. So, there is no physical gravitational interactions in space-time and space-time is curved if gravitational field does not vanish. In this case, the space-time metric is not Minkowski metric, but $g^{\alpha\beta}$ (or $g_{\alpha\beta}$). In other words, Minkowski metric is the space-time metric if we discuss gravity in physical representation while metric tensor $g^{\alpha\beta}$ (or $g_{\alpha\beta}$) is space-time metric if we discuss gravity in geometrical representation. So, if we use Minkowski metric in our discussion, it means that we are in physical representation of gravity; if we use metric tensor $g^{\alpha\beta}$ (or $g_{\alpha\beta}$) in our discussion, it means that we are in geometrical representation.

In one representation, gravity is treated as physical interactions, while in another representation, gravity is treated as geometry of space-time. But we know that there is only one physics for gravity, so two representations of gravity must be equivalent. This equivalence means that the nature of gravity is physics-geometry duality, i.e., gravity is a kind of physical interactions which has the characteristics of geometry; it is also a geometry of space-time which is essentially a kind of physical interactions. Now, let's go into the geometrical representation of gravity and use $g^{\alpha\beta}$ and $g_{\alpha\beta}$ as space-time metric tensors. In this picture, we can obtain an Einstein-like field equation with cosmological constant.

Define

$$\Lambda \triangleq \frac{1}{2}(R + 4g^2\mathcal{L}_0). \tag{5.1}$$

Then action given by eq.(3.34) will be changed into

$$S = S_g + S_M, \tag{5.2}$$

where

$$S_g = \frac{-1}{16\pi G} \int \mathrm{d}^4x \sqrt{-g}(R - 2\Lambda), \tag{5.3}$$

$$S_M = \int \mathrm{d}^4x \mathcal{L}_M, \tag{5.4}$$

where G is the Newtonian gravitational constant, which is given by

$$G = \frac{g^2}{4\pi}, \tag{5.5}$$

R is the scalar curvature, Λ is the cosmological constant, $\mathcal{L}_M$ is the lagrangian density for matter fields. Scalar curvature R can be expressed by gravitational gauge field C_μ^α[18]. We have added the action for matter fields into eq.(5.2) and denoted the action for pure gravitational gauge field as S_g. Using the following relations

$$\delta\sqrt{-g} = \frac{1}{2}\sqrt{-g} g^{\mu\nu} \delta g_{\mu\nu}, \tag{5.6}$$

$$\sqrt{-g} g^{\mu\nu} \delta R_{\mu\nu} = \partial_\lambda W^\lambda, \tag{5.7}$$

$$T_m^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta g_{\mu\nu}(x)}, \tag{5.8}$$

where $T_m^{\mu\nu}$ is the energy-momentum tensor of matter fields and W^λ is a contravariant vector, we can obtain the Einstein's field equation with cosmological constant Λ,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}, \tag{5.9}$$

where $T_{\mu\nu}$ is the revised energy-momentum tensor, whose definition is

$$T_{\mu\nu} \triangleq T_{m\mu\nu} - \frac{1}{4\pi G} \frac{\delta\Lambda}{\delta g^{\mu\nu}}. \tag{5.10}$$

In eq.(5.10), the difinition of $\frac{\delta\Lambda}{\delta g^{\mu\nu}}$ is not clear, because Λ is a function of G_μ^α, not a function of $g^{\mu\nu}$. So, we need to give out an explicite definition of $\frac{\delta\Lambda}{\delta g^{\mu\nu}}$. According to eq.(3.13), we have

$$\frac{\partial g^{\mu\nu}}{\partial G_\alpha^\lambda} = \delta_\lambda^\mu \eta^{\alpha\beta} G_\beta^\nu + \delta_\lambda^\nu \eta^{\alpha\beta} G_\beta^\mu. \tag{5.11}$$

Therefore, we have

$$\frac{\delta\Lambda}{\delta G_\alpha^\lambda} = 2\eta^{\alpha\beta} G_\beta^\nu \frac{\delta\Lambda}{\delta g^{\lambda\nu}} \tag{5.12}$$

It gives out

$$\frac{\delta \Lambda}{\delta g^{\mu\nu}} = \frac{1}{4} \eta_{\alpha\beta} \left(G^{-1\beta}_{\nu} \frac{\delta \Lambda}{\delta G^{\mu}_{\alpha}} + G^{-1\beta}_{\mu} \frac{\delta \Lambda}{\delta G^{\nu}_{\alpha}} \right), \tag{5.13}$$

which gives out the explicite expression for $\frac{\delta \Lambda}{\delta g^{\mu\nu}}$. Eq.(5.9) is quite like the Einstein's field equation with cosmological constant in form, so we call it the Einstein-like field equation with cosmological constant.

The explicite expression of $\mathcal{L}_0$ is given by eq.(3.28), and the explicite expression of scalar curvature R can be found in [18]. According to eq.(5.1), the explicate formulation of Λ can be calculated. In other words, the cosmological constant can be expressed in terms of gravitational gauge field C^{α}_{μ}. The we make large scale average of it which will gives out the average value of the cosmological constant. A rough estimation shows that[21]

$$\Lambda \sim 2.92 \times 10^{-52} m^{-2}, \tag{5.14}$$

which is quite close to experimental results[22],

$$\Lambda = 3.51 \times 10^{-52} \Omega_{\Lambda} h_0^2 m^{-2}, \tag{5.15}$$

with Ω_{Λ} is the scaled cosmological constant and h_0 is the normalized Hubble expansion rate, whose values are

$$-1 < \Omega_{\Lambda} < 2, \tag{5.16}$$

$$0.6 < h_0 < 0.8. \tag{5.17}$$

6 Classical Tests

It is known that General Relativity is tested by three main classical tests, which are perihelion procession, deflection of light and gravitational red-shift. All these three tests are related to geodesic curve equation and schwarzachild solution in general relativity. If we know geodesic curve equation and space-time metric, we can calculate perihelion procession, deflection of light and gravitational red-shift. In this chapter, we discuss this problem from the point of view of gauge theory of gravity.

In order to discuss classical tests of gravity, for the sake of convience, we use the geometrical representation of gravity. As we have stated above, in the geometrical representation of gravity, gravity is not treated as physical interactions in space-time. In the gemetrical representation of gravity, we use the same manner which is widely used in general relativity to discuss classical tests and to explain the predictions with observations. In the geometrical representation of gravity, if there is no other physical interactions in space time, any mass point can not feel any physical forces when it moves in space-time. So, it must move along the curve which has the least length. Suppose that a particle is moving from point A to point B along a curve. Define

$$T_{BA} = \int_A^B \sqrt{-g_{\mu\nu}\frac{\mathrm{d}x^\mu}{\mathrm{d}p}\frac{\mathrm{d}x^\nu}{\mathrm{d}p}}\mathrm{d}p, \tag{6.1}$$

where p is a parameter that describe the orbit. The real curve that the particle moving along corresponds to the minimum of T_{AB}. The minimum of T_{AB} gives out the following geodesic curve equation

$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}p^2} + \Gamma^\mu_{\nu\lambda}\frac{\mathrm{d}x^\nu}{\mathrm{d}p}\frac{\mathrm{d}x^\lambda}{\mathrm{d}p} = 0, \tag{6.2}$$

where $\Gamma^\mu_{\nu\lambda}$ is the affine connection

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda\sigma}\left(\frac{\partial g_{\mu\sigma}}{\partial x^\nu} + \frac{\partial g_{\nu\sigma}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\sigma}\right). \tag{6.3}$$

Eq.(6.2) gives out the curve that a free particle moves along in curved space-time if we discuss physics in the geometrical representation of gravity.

Now, we need to calculate a schwarzchild-like solution in gauge theory of gravity. In chapter 4, we have obtained a solution of C^α_μ for static spherically symmetric gravitational fields in linear approximation of gC^α_μ. But expetimental tests, especially perihelion procession, are sensitive to second order of gC^α_μ. The best way to do this is to solve the equation of motion of gravitational gauge field in the second order approximation of gC^α_μ. But this equation of motion is a non-linear second order partial differential equations. It is rather difficult to solve them. So, we had to find some other method to do this. The perturbation method is used to do this. After considering corrections from gravitational energy of the sun in vacuum space and gravitational interactive energy between the sun and the Mercury, the equivalent gravitational gauge field in the second order approximation is[23]

$$gC^0_0 = -\frac{GM}{r} - \frac{3}{2}\frac{G^2M^2}{r^2} + O\left(\frac{G^3M^3}{r^3}\right). \tag{6.4}$$

Then using eq.(3.14), we can obtain the following solution

$$
\begin{aligned}
d\tau^2 \;\; = \;\; & \left[1 - \frac{2GM}{r} + O\left(\frac{G^3M^3}{r^3}\right)\right] dt^2 - \left[1 + \frac{2GM}{r} + O\left(\frac{G^2M^2}{r^2}\right)\right] dr^2 \\
& -r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2,
\end{aligned}
\tag{6.5}
$$

where we have use the following gauge for gravitational gauge field,

$$
C^{\mu}_{\mu} = 0. \tag{6.6}
$$

This solution is quite similar to schwarzschild solution, but it is not schwarzschild solution, so we call it schwarzschild-like solution. If we use Eddington-Robertson expansion, we will find that for the present schwarzschild-like solution[24],

$$
\alpha = \beta = \gamma = 1. \tag{6.7}
$$

They have the same values as schwarzschild solution in general relativity and all three tests are only sensitive to these three parameters, so gauge theory of gravity gives out the same theoretical predictions as those of general relativity[23]. More detailed discussions on classical tests can be found in literature [23]. (This result hold for those models which have other choice of η_2 and $J(C)$ which is duscussed in [18].)

7 Summary and Comments

In this paper, we have discussed a completely new quantum gauge theory of gravity. Finally, we give a simple summary to the whole theory. 1) In leading order approximation, the gravitational gauge field theory gives out classical Newton's theory of gravity. 2) It gives out Einstein's field equation with cosmological constant. 3) Gravitational gauge field theory is a renormalizable quantum theory. 4) It gives out the same theoretical predictions on three main classical tests as those of general relativity. 5) It can also predict the theoretical value of cosmological constant.

In one point of view, gravity is treated as physical interactions in flat space-time. In this case, the physical space-time is always flat. In another point of view, gravity is put into the structure of space-time and space-time is curved if the gravitational gauge field does not vanish. Two points of view are based on different space-time, i.e., the first point of view is based on physical space-time, and the second point of view is based on gravitational gauge group space-time. Two-points of view should give out the same physics, because there is only one physics for gravity. In other wards, they are equivalent. In the first point of view, gravity is a kind of physical interactions, and in the second point of view, gravity is the geometry of space-time. Because they are equivalent, the nature of gravity is physics-geometry duality, i.e., gravity is a kind of physical interactions which has the characteristics of geometry, it is also a geometry of space-time which is essentially a kind of physical interactions.

In general relativity, geometrical nature of gravity is the result of equivalence principle, but equivalence principle is not the necessary condition of the geometrical nature of gravity. From mathematical point of view, the geometrical nature of gravity in gauge theory of gravity originates from of geometrical nature of gravitational gauge transformation, i.e., the geometrical nature of translation transformation.

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